Physics of Elastic Spheres Skipping on Water

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Abstract

It is well known that one can skip a stone across the water surface, but less well known that a ball can also be skipped on water. The Waboba\(^\circledR\) is an elastic ball used in a game of aquatic keep away in which players pass the ball by skipping it along the water. We investigate the physics of skipping elastic balls to elucidate the mechanisms by which they bounce off water. High speed video reveals that, upon impact with the water, the ball creates a cavity and deforms significantly due to the hyperelasticity of the material; the flattened spheres resemble skipping stones. With an increased wetted surface area, a large hydrodynamic lift force is generated causing the ball to launch back into the air, onward to an awaiting Wabobian. Unlike stone skipping, the elasticity of the ball plays an important roll in determining the success of the skip. Through experimentation, we demonstrate that a deformation timescale during impact must be longer than the collision time in order to achieve a successful skip. Scaling for the deformation time is presented. Aided by a numerical model, we identify two dominate modes of sphere deformation. The effect of impact velocity and angle on the skipping behavior is also investigated. We hope that our findings lead to an optimized Waboba experience such that the painful alternative game of “skipping stone keep away” finally becomes obsolete.

1 Introduction

Perhaps the most obvious and enjoyable example of object skipping on water is that of stone skipping \([1, 2]\). Rosellini et al. \([2]\) studied the physics of stone skipping to reveal an optimal attack angle for a stone throw. A significant amount of work also exists pertaining to water entry of rigid projectiles at shallow angles to the surface (e.g., \([3, 4, 5, 6]\)). Beyond just stones, spheres can also be skipped on water \([7, 8]\). To achieve skipping with rigid spheres the impact angle must be less than a critical angle given by

\[
\theta_c = \frac{18}{\sqrt{\gamma}}
\]  

where \(\gamma\) is the specific gravity of the sphere material. However, one may ask what happens when the object - in this case a sphere - is highly deformable?

In this paper the impacting bodies are hyperelastic balls used in a game of aquatic keep away. Figure 1 shows one such ball thrown at the surface of a lake. The sphere skips an astonishing 23 times before landing on the opposite shore of the lake approximately 46 m away (Figure 1(h)), begging the question: what are the physical mechanisms that allow this ball to skip so magnificently on water?

We investigate the influence of impact angle and velocity on the skipping mechanics and show that large nonlinear deformations play a significant role in the skipping. The success of skipping is shown to depend on the interplay between two timescales: the collision time and the deformation time; the effect of experimental parameters on each is explored. We also identify the minimum velocity required to achieve skipping as a function of impact angle.

2 Methods

The experimental setup is depicted in Figure 2 and consisted of a gas powered launcher mounted above the waterline of a long tank (W x H x L = 1.17 m x 1.02 m x 25.9 m). The tank was filled with water to depth of 0.457 m. A custom barrel made for each sphere type with inner diameter (ID) slightly larger than the sphere diameter was attached to the launcher. The location of the first impact remained the same for all tests and was 22.3 m from the far end of the tank. Three NAC Image Technology GX-3 high speed cameras imaged the first impact, which was illuminated by diffuse white backlight. Impact angle and velocity were measured directly from the high speed images. Four additional NAC Image Technology GX-3 high speed cameras were arranged in various locations to capture the number of skips as well as other details about the skipping spheres downrange.

Three different sphere types were launched at a range of velocities at four different angles. The mean value of angles tested were \(10.2^\circ\), \(20.8^\circ\), \(30.3^\circ\) and \(40.8^\circ\) with a maximum deviation of \(2.16^\circ\) across all angles.

Table 1 lists the radius \(R\), mass ratio \((m^* = m_s/m_w)\)
and elastic modulus $E$ for each sphere tested. In lieu of published material properties for the spheres (which is proprietary information), we performed a force-displacement measurement on an Instron® material testing machine to infer the elastic modulus. Puttock & Thwaite [9] presented an analytical solution for the force-displacement ($F - d$) relationship for a linear elastic sphere compressed between two parallel infinite plates, which is given by

$$F = \frac{E\sqrt{2R}}{3(1-\sigma^2)}d^{3/2} \tag{2}$$

where $\sigma$ is Poisson’s ratio, which we assume to be equal to 0.5. Using a non-linear minimization technique, the value of $E$ of each sphere was inferred by fitting Equation 2 to the $F - d$ data for normalized deformation $d/D \leq 0.3$, where $D$ is the sphere diameter. We expected the materials to deviate from linear elastic behavior and thus fit to a region of relatively small deformation despite that fact that the deformations observed in the skipping experiments are sometimes larger. Figure 3 shows the force-displacement data and theoretical fit for sphere 3; the theoretical fit is extrapolated beyond $d/D = 0.3$ showing that the actual sphere behavior is not linear elastic at larger strains. The trends are the same for the other two spheres. Work is ongoing to numerically model the nonlinear force-displacement behavior of spheres using the Finite Element Analysis (FEA) software Abaqus/Explicit to better describe the material model.

In addition to serving as a tool for determining a nonlinear material model, the Abaqus software was used to simulate the sphere skipping event. The numerical model provided additional insight into the skipping events, and in the future will be used to simulate tests that would be difficult to perform experimentally. The code employed an explicit time integration solver suitable for highly nonlinear contact problems [10].

The modeling technique used the Coupled Eulerian-Lagrangian (CEL) approach where the elastic spheres were modeled in a Lagrangian frame and the fluid in an Eulerian frame. The Eulerian-Lagrangian contact formulation is based on an enhanced immersed boundary method, which automatically computes and tracks the interface between the Lagrangian structure and the Eulerian materials. Contact forces are computed using a penalty method, which introduces a “spring” force proportional to the amount of surface penetration. This artificial contact force is usually small, and does not influence the general behavior of the model.

<table>
<thead>
<tr>
<th>Sphere</th>
<th>$R$ (mm)</th>
<th>$m^* = m_s/m_w$</th>
<th>$E$ (N/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28.5</td>
<td>0.873</td>
<td>6.39E+04</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>0.967</td>
<td>7.21E+04</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>0.772</td>
<td>7.77E+04</td>
</tr>
</tbody>
</table>
The mesh contained a domain of Eulerian elements, some of which were initially void. The boundaries of the domain were idealized as infinite, which was appropriate for the relatively small size of the numerical model compared to the test cases. The pressure field in the water was evaluated using the linear Us-Up Hugoniot form of the Mie-Gruneisen equation of state. The elastic spheres were modeled as a hyperelastic vulcanized rubber; the finite-strain material model is isotropic, nonlinear, and is valid for large strains.

3 Results & Discussion

The experimental and numerical studies were aimed at understanding the mechanisms responsible for the impressive sphere skipping and identifying the role of impact angle and velocity in the skipping behavior.

3.1 Skipping Mechanisms

Figure 4 shows the initial time instants of the first impact of sphere 3 from the impact angle $\beta = 30.3^\circ$ tests; the impact velocity is $U = 25.2$ m/s. The sphere deforms by an amount of $d/D = 0.6$ in just 4 ms.

Figure 5(a) shows a time sequence for the first skip of sphere 1 with $\beta = 26.2^\circ$ and $U = 8.6$ m/s. Note that this particular test was not used in the study of behavior as a function of impact angle as $\beta$ deviated too far from the mean angle groupings. Nonetheless, the sequence shows representative behavior of a relatively low speed skip in which the sphere deforms, creates a cavity and then somehow again impacts its own cavity prior to leaving the surface. The second cavity impact is evidenced by another cavity beginning to form at $t = 37$ ms ($t = 0$ ms corresponds to the moment of first contact with the water). When the sphere impacts its own cavity in this manner, we define this event as a failed skip as energy is lost during the second impact and the sphere never skips more than once.

Figure 5(b) shows a sequence from the numerical model forms significantly producing a much larger contact area compared to a rigid sphere of equivalent diameter. Furthermore, by assuming a shape similar to a circular disk, the force coefficient becomes much larger. This combined with the increased surface area produces a larger hydrodynamic force compared to a rigid sphere. This explains why these elastic spheres skip more successfully than rigid spheres. In fact, applying Equation 1 for sphere 3 predicts a critical angle of $20.5^\circ$, but we observe these spheres to skip at impact angles over $40^\circ$.

Figure 5(a) shows a time sequence for the first skip of sphere 1 with $\beta = 26.2^\circ$ and $U = 8.6$ m/s. Note that this particular test was not used in the study of behavior as a function of impact angle as $\beta$ deviated too far from the mean angle groupings. Nonetheless, the sequence shows representative behavior of a relatively low speed skip in which the sphere deforms, creates a cavity and then somehow again impacts its own cavity prior to leaving the surface. The second cavity impact is evidenced by another cavity beginning to form at $t = 37$ ms ($t = 0$ ms corresponds to the moment of first contact with the water). When the sphere impacts its own cavity in this manner, we define this event as a failed skip as energy is lost during the second impact and the sphere never skips more than once.

Figure 5(b) shows a sequence from the numerical model.
3.2 Skipping Regimes

The influence of impact velocity on the behavior observed in the first skip is now discussed. Figure 6 shows time sequences of the first skip for sphere 3 from the $\beta = 30.3^\circ$ tests with five different impact velocities: (a) $U = 8.08$ m/s, (b) $U = 12.33$ m/s, (c) $U = 16.87$ m/s, (d) $U = 22.2$ m/s, (e) $U = 25.2$ m/s. The two lowest speed cases demonstrate the failed skip behavior whereby a stress wave propagating around the periphery of the sphere impacts the cavity (multiple times for these two tests). In case (a), the sphere barely has enough energy to leave the surface. Tests (d) & (e) are also successful skips, but the sphere exhibits a different mode of vibration than the counter clockwise stress wave previously observed. Figure 7 depicts a conceptual (albeit simplified) period of the vibration mode whereby the sphere is compressed into an elliptical shape. Time $t_0$ corresponds to the time at which the sphere is maximally compressed during the impact and $T$ is the period of oscillation of the vibration mode.

3.3 Critical Timescales

The phenomena represented in Figures 5 & 6 suggest two critical timescales related to the elastic sphere skipping event: one related to duration in which the sphere is in contact...
with the water and one related to the deformation of the sphere. The former is called the collision time, $t_c$, and is defined simply as the time in which the sphere is in contact with the water. The latter is called the wave time, $t_w$, and is defined in one of two ways depending on the deformation regime. For the lower speed regime in which a stress wave propagates around the sphere periphery, we define $t_w = t_0 + \tau_d$, where $t_0$ is the time elapsed from initial sphere contact to the time the stress wave is initiated. The deformation time $\tau_d$ is time between stress wave initiation and stress wave impact with either the cavity (Figure 6(a) & (b)) or the splash curtain (Figure 6(c)). Similarly, when the sphere deforms in the mode shown in Figure 6(d) & (e), $t_w$ is defined as $t_0 + \tau_d$, where $t_0$ is the time elapsed from initial contact with the water to the time the sphere is maximally compressed and $\tau_d$ is defined as $T/2$.

Figure 8 plots $t_c$ and $t_w$ as a function of $U$ for sphere 3, $\beta = 30.3^\circ$ tests. The red circle denotes the point at which the collision time equals the wave time, which marks the minimum impact speed required for successful skipping. In the same way, the minimum speed is found for the other angles for sphere 3 and plotted in Figure 9. The data reflect the intuition that one must throw the sphere harder to achieve a skip at higher angles, and the relationship between minimum speed and impact angle is nonlinear.

Finally, we aim to develop a scaling for $\tau_d$ for cases that demonstrate the peripherally propagating stress wave. One might expect both the elasticity and impact speed to play a role in the deformation timescale. Therefore, Figure 10 plots the dimensionless deformation time $\tau_d^* = \tau_d U_y / y$ versus the dimensionless parameter $E/\rho_w U_y^2$, where $U_y$ is...
the vertical component of impact velocity and $\rho_w$ is the density of water. The data demonstrate excellent collapse indicating that the deformation time can be predicted for various material properties and impact conditions. Ongoing work includes development of a suitable scaling for the collision time such that success of skipping can be predicted based on sphere properties and impact conditions.

4 Conclusions

The physics of elastic spheres that skip on water was explored. High speed video revealed that the spheres deform significantly and rapidly upon water impact, creating a large contact area and thus large hydrodynamic force compared to a rigid sphere of equal diameter. Through experimental and numerical study, two modes of deformation were identified. One mode occurs at high speeds and the other occurs for lower impact speeds and results in a wave that propagates around the periphery of the sphere. If the wave time $t_w = t_0 + \tau_d$ is shorter than the collision time, the wave will impact the cavity before the sphere leaves the surface resulting in what we define as a failed skip. The dimensionless deformation time $\tau_d$ for the lower speed deformation mode was shown to scale with a ratio of elasticity to inertia, $E/\rho_w U^2_y$. In future work a scaling will be sought for the collision time and the physics of the first skip will be related to the total number of skips.

References